IV Semester M.Sc. Degree Examination, June 2017
(RNS Scheme) (Repeaters)
MATHEMATICS
M-403(A) : Graph Theory

Time : 3 Hours   Max. Marks : 80

Instructions : *1) Answer* ***any five*** *question.*
2) ***Each*** *question carry* ***equal*** *marks.*

1. a) Define :
   i) Vertex connectivity
   ii) Edge connectivity of a graph G.
   Show that $\lambda(K_n) = n - 1$. **6**

   b) For a cubic graph G, prove that $K(G) = \lambda(G)$. **6**

   c) Define n-connected graph. If G(p, q) is a n-connected graph, then prove that $q \geq \left[\frac{pn}{2}\right]$. **4**

2. a) Let G be a planar graph with p-vertices, q-edges, r-regions and k-components. Then prove that p – q + r = k + 1. **5**

   b) Show that there are only five regular polyhedra. **7**

   c) Prove that K_5 and $K_{3, 3}$ are non-planar. **4**

3. a) Define outer planar graph. Prove that every maximal outer planar graph G with p-points has
   i) 2p – 3 edges
   ii) atleast 3 points of degree not exceeding 3. **5**

   b) If G is a maximal planar (p, q) graph with $p \geq 3$, then prove that q = 3p – 6. **6**

   c) Define crossing number of a graph. Determine the crossing number of K_6. **5**

4. a) Prove that for any graph G, $\frac{p^2}{p^2 - 2q} \le \chi(G) \le \Delta(G) + 1$. **7**

b) Prove that for any graph G, the sum and product $\chi(G)$ and $\chi(\overline{G})$ satisfy the inequalities.

i) $2(p)^{1/2} \le \chi(G) + \chi(\overline{G}) \le p + 1$

ii) $p \le \chi(G)\,\chi(\overline{G}) < \left(\frac{p+1}{2}\right)^2$. **7**

c) Write a short note on four color conjecture. **2**

5. a) Show that for any graph G with p-vertices is a tree if and only if $f(G, \lambda) = \lambda(\lambda - 1)^{p-1}$. Also find $f(K_p, \lambda)$ and $f(\overline{K_p}, \lambda)$. **7**

b) Show that every planar graph can be properly colored with five colors. **6**

c) Define chromatic polynomial of a graph. Determine the chromatic polynomial of

i) K_5

ii) C_9

iii) $K_{4,5}$. **3**

6. a) Define :

i) Line graph

ii) Total graph

iii) Block graph with an example.

Further show that $K_{1,3}$ is not a line graph. **8**

b) Draw all tournaments with four vertices. **4**

c) Prove that every tournament has a spanning path. **4**

7. a) Let X be the adjacency matrix of a simple graph G, then prove that the $(i, j)^{th}$ entry in X^n in the number of edge sequence of n-edges between the vertices V_i and V_j. **5**

b) Let A be an incidence matrix of a disconnected graph with p-vertices and k-components. Then prove that the rank of A is (p – k). **5**

c) Let B and A be the circuit matrix and the incidence matrix, respectively, of a simple graph whose columns are arranged using the same order of edges, then prove that $A \cdot B^T = B \cdot A^T \equiv 0 \pmod 2$. **6**

8. a) Show that a dominating set D is a minimal dominating set if and only if each vertex $u \in d$ one of the following two conditions holds :

   i) u is an isolated of D.

   ii) There exists a vertex $v \in V - D$ for which $N(v) \cap D = \{u\}$. **8**

b) Prove that for any graph G,

   i) $p - q \le \gamma(G) \le p - B_0(G)$

   ii) $\left[\frac{p}{\Delta + 1}\right] \le \gamma(G) \le p - \Delta$. **8**
